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Supersymmetry and DLCQ Limit of Lie 3-algebra Model of M-theory

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Abstract

In arXiv:1003.4694, we proposed two models of M-theory, Hermitian 3-algebra model and Lie 3-algebra model. In this paper, we study the Lie 3-algebra model with a Lorentzian Lie 3-algebra. This model is ghost-free despite the Lorentzian 3-algebra. We show that our model satisfies two criteria as a model of M-theory. First, we show that the model possesses $\mathcal{N} = 1$ supersymmetry in eleven dimensions. Second, we show the model reduces to BFSS matrix theory with finite size matrices in a DLCQ limit.

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1 Introduction

BFSS matrix theory [1] is one of the strong candidates of non-perturbative definition of superstring theory. It is conjectured to describe infinite momentum frame (IMF) limit of M-theory and many evidences were found. Because only D0-branes in type IIA superstring theory survive in this limit, BFSS matrix theory is defined by the one-dimensional maximally supersymmetric Yang-Mills theory. Since the theory is a gauge theory, a matrix representation is allowed and dynamics of a many-body system can be described by using diagonal blocks of matrices. However, it seems impossible to derive full dynamics of M-theory from BFSS matrix theory because it treats D0-branes as fundamental degrees of freedom. For example, we do not know the manner to describe longitudinal momentum transfer of D0-branes. Therefore, we need a matrix model that treats membranes as fundamental degrees of freedom in order to derive full dynamics of M-theory.

IIB matrix model [2] is also one of the strong candidates of non-perturbative definition of superstring theory. It starts with the Green-Schwartz type IIB superstring action in order to treat strings themselves as fundamental degrees of freedom. If we fix the κ symmetry to Schild gauge $\theta_1 = \theta_2$, the action reduces to that of the zero-dimensional maximal supersymmetric Yang-Mills theory with area preserving diffeomorphism (APD) symmetry. Since the resultant action is a gauge theory, it describes dynamics of many-body systems. IIB matrix model is defined by replacing the APD algebra with $u(N)$ Lie algebra in the action.

In paper [3], we obtained matrix models of M-theory in an analogous way to obtain IIB matrix model. We started with the Green-Schwartz supermembrane action in order to obtain matrix models of M-theory that treat membranes themselves as fundamental degrees of freedom. We showed, by using an approximation, that the action reduces to that of a zero-dimensional gauge theory with volume preserving diffeomorphism (VPD) symmetry [4, 5] if we fix the κ symmetry of the action to a semi-light-cone gauge, $\Gamma_{012}\Psi = \Psi$. We proposed two 3-algebra models of M-theory which are defined by replacing VPD algebra with finite-dimensional 3-algebras in the action. Because the 3-algebra models are gauge theories, they are expected to describe dynamics of many-body systems as in the other matrix models.

One of the two models is based on Hermitian 3-algebra [6–9] (Hermitian 3-algebra model), whereas the another is based on Lie 3-algebra [10–22] (Lie 3-algebra model). The Hermitian 3-algebra model with $u(N) \oplus u(N)$ symmetry was shown to reduce to BFSS matrix theory

with finite size matrices when a DLCQ limit is taken in [3]. A supersymmetric deformation of the Lie 3-algebra model with the \mathcal{A}_4 algebra was investigated by adding mass and flux terms in [23].

In this paper, we study the Lie 3-algebra model with a Lorentzian Lie 3-algebra. We show that this model satisfies two criterion as a model of M-theory. In section two, we show that the model possesses $\mathcal{N} = 1$ supersymmetry in eleven dimensions. In section three, we show the model reduces to BFSS matrix theory with finite size matrices in a DLCQ limit as it should do: it is generally shown that M-theory reduces to such BFSS matrix theory in a DLCQ limit [24–27].

2 $\mathcal{N} = 1$ Supersymmetry Algebra in Eleven Dimensions

In [3], we proposed the Lie 3-algebra model of M-theory, whose action is given by

$$\begin{aligned} S_0 = & \left\langle -\frac{1}{12}[X^I, X^J, X^K]^2 - \frac{1}{2}(A_{\mu ab}[T^a, T^b, X^I])^2 \right. \\ & - \frac{1}{3}E^{\mu\nu\lambda}A_{\mu ab}A_{\nu cd}A_{\lambda ef}[T^a, T^c, T^d][T^b, T^e, T^f] \\ & \left. - \frac{i}{2}\bar{\Psi}\Gamma^\mu A_{\mu ab}[T^a, T^b, \Psi] + \frac{i}{4}\bar{\Psi}\Gamma_{IJ}[X^I, X^J, \Psi] \right\rangle. \end{aligned} \quad (2.1)$$

The fields are spanned by Lie 3-algebra T^a as $X^I = X_a^I T^a$, $\Psi = \Psi_a T^a$ and $A^\mu = A_{ab}^\mu T^a \otimes T^b$, where $I = 3, \dots, 10$ and $\mu = 0, 1, 2$. $\langle \rangle$ represents a metric for the 3-algebra. Ψ is a Majorana spinor of $SO(1,10)$ that satisfies $\Gamma_{012}\Psi = \Psi$. $E^{\mu\nu\lambda}$ is a Levi-Civita symbol in three-dimensions. In this section, we will show that this action possesses $\mathcal{N} = 1$ supersymmetry in eleven-dimensions.

The action is invariant under 16 dynamical supersymmetry transformations,

$$\begin{aligned} \delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\ \delta A_{\mu ab}[T^a, T^b, \] &= i\bar{\epsilon}\Gamma_\mu\Gamma_I[X^I, \Psi, \] \\ \delta\Psi &= -A_{\mu ab}[T^a, T^b, X^I]\Gamma^\mu\Gamma_I\epsilon - \frac{1}{6}[X^I, X^J, X^K]\Gamma_{IJK}\epsilon, \end{aligned} \quad (2.2)$$

where $\Gamma_{012}\epsilon = -\epsilon$. These supersymmetries close into gauge transformations on-shell,

$$\begin{aligned}
[\delta_1, \delta_2]X^I &= \Lambda_{cd}[T^c, T^d, X^I] \\
[\delta_1, \delta_2]A_{\mu ab}[T^a, T^b, \quad] &= \Lambda_{ab}[T^a, T^b, A_{\mu cd}[T^c, T^d, \quad]] - A_{\mu ab}[T^a, T^b, \Lambda_{cd}[T^c, T^d, \quad]] + 2i\bar{\epsilon}_2\Gamma^\nu\epsilon_1 O_{\mu\nu}^A \\
[\delta_1, \delta_2]\Psi &= \Lambda_{cd}[T^c, T^d, \Psi] + (i\bar{\epsilon}_2\Gamma^\mu\epsilon_1\Gamma_\mu - \frac{i}{4}\bar{\epsilon}_2\Gamma^{KL}\epsilon_1\Gamma_{KL})O^\Psi,
\end{aligned} \tag{2.3}$$

where gauge parameters are given by $\Lambda_{ab} = 2i\bar{\epsilon}_2\Gamma^\mu\epsilon_1 A_{\mu ab} - i\bar{\epsilon}_2\Gamma_{JK}\epsilon_1 X_a^J X_b^K$. $O_{\mu\nu}^A = 0$ and $O^\Psi = 0$ are equations of motions of $A_{\mu\nu}$ and Ψ , respectively, where

$$\begin{aligned}
O_{\mu\nu}^A &= A_{\mu ab}[T^a, T^b, A_{\nu cd}[T^c, T^d, \quad]] - A_{\nu ab}[T^a, T^b, A_{\mu cd}[T^c, T^d, \quad]] \\
&\quad + E_{\mu\nu\lambda}(-[X^I, A_{ab}^\lambda[T^a, T^b, X_I], \quad] + \frac{i}{2}[\bar{\Psi}, \Gamma^\lambda\Psi, \quad]) \\
O^\Psi &= -\Gamma^\mu A_{\mu ab}[T^a, T^b, \Psi] + \frac{1}{2}\Gamma_{IJ}[X^I, X^J, \Psi].
\end{aligned} \tag{2.4}$$

(2.3) implies that a commutation relation between the dynamical supersymmetry transformations is

$$\delta_2\delta_1 - \delta_1\delta_2 = 0, \tag{2.5}$$

up to the equations of motions and the gauge transformations.

Lie 3-algebra with an invariant metric is classified into four-dimensional Euclidean \mathcal{A}_4 algebra and Lie 3-algebras with indefinite metrics in [16–18, 28, 29]. We do not choose \mathcal{A}_4 algebra because its degrees of freedom are just four. We need an algebra with arbitrary dimensions N , which is taken to infinity to define M-theory. Here we choose the most simple indefinite metric Lie 3-algebra, so called Lorentzian Lie 3-algebra associated with $u(N)$ Lie algebra,

$$\begin{aligned}
[T^{-1}, T^a, T^b] &= 0 \\
[T^0, T^i, T^j] &= [T^i, T^j] = f^{ij}{}_k T^k \\
[T^i, T^j, T^k] &= f^{ijk} T^{-1},
\end{aligned} \tag{2.6}$$

where $a = -1, 0, i$ ($i = 1, \dots, N^2$). T^i are generators of $u(N)$. A metric is defined by a symmetric bilinear form,

$$\langle T^{-1}, T^0 \rangle = -1 \tag{2.7}$$

$$\langle T^i, T^j \rangle = h^{ij}, \tag{2.8}$$

and the other components are 0. The action is decomposed as

$$S = \text{Tr} \left(-\frac{1}{4} (x_0^K)^2 [x^I, x^J]^2 + \frac{1}{2} (x_0^I [x_I, x^J])^2 - \frac{1}{2} (x_0^I b_\mu + [a_\mu, x^I])^2 - \frac{1}{2} E^{\mu\nu\lambda} b_\mu [a_\nu, a_\lambda] \right. \\ \left. + i\bar{\psi}_0 \Gamma^\mu b_\mu \psi - \frac{i}{2} \bar{\psi} \Gamma^\mu [a_\mu, \psi] + \frac{i}{2} x_0^I \bar{\psi} \Gamma_{IJ} [x^J, \psi] - \frac{i}{2} \bar{\psi}_0 \Gamma_{IJ} [x^I, x^J] \psi \right), \quad (2.9)$$

where we have renamed $X_0^I \rightarrow x_0^I$, $X_i^I T^i \rightarrow x^I$, $\Psi_0 \rightarrow \psi_0$, $\Psi_i T^i \rightarrow \psi$, $2A_{\mu 0i} T^i \rightarrow a_\mu$, and $A_{\mu ij} [T^i, T^j] \rightarrow b_\mu$. In this action, T^{-1} mode; X_{-1}^I , Ψ_{-1} or A_{-1a}^μ does not appear, that is they are unphysical modes. Therefore, the indefinite part of the metric (2.7) does not exist in the action and our model is ghost-free like a model in [30]. This action can be obtained by a dimensional reduction of the three-dimensional $\mathcal{N} = 8$ BLG model [13–15] with the same 3-algebra. The BLG model possesses a ghost mode because of its kinetic terms with indefinite signature. On the other hand, our model does not possess a kinetic term because it is defined as a zero-dimensional field theory like IIB matrix model [2].

This action is invariant under the translation

$$\delta x^I = \eta^I, \quad \delta a^\mu = \eta^\mu, \quad (2.10)$$

where η^I and η^μ belong to $u(1)$. This implies that eigen values of x^I and a^μ represent an eleven-dimensional space-time.

The action is also invariant under 16 kinematical supersymmetry transformations

$$\tilde{\delta}\psi = \tilde{\epsilon}, \quad (2.11)$$

and the other fields are not transformed. $\tilde{\epsilon}$ belong to $u(1)$ and satisfy $\Gamma_{012}\tilde{\epsilon} = \tilde{\epsilon}$. $\tilde{\epsilon}$ and ϵ should come from 16 components of 32 $\mathcal{N} = 1$ supersymmetry parameters in eleven dimensions, corresponding to eigen values ± 1 of Γ_{012} , respectively, as in the case of the semi-light-cone supermembrane. Its target-space $\mathcal{N} = 1$ supersymmetry consists of remaining 16 target-space supersymmetries and transmuted 16 κ -symmetries in the semi-light-cone gauge, $\Gamma_{012}\Psi = \Psi$ [3, 31, 32].

A commutation relation between the kinematical supersymmetry transformations is given by

$$\tilde{\delta}_2 \tilde{\delta}_1 - \tilde{\delta}_1 \tilde{\delta}_2 = 0. \quad (2.12)$$

The 16 dynamical supersymmetry transformations (2.2) are decomposed as

$$\begin{aligned}
\delta x^I &= i\bar{\epsilon}\Gamma^I\psi \\
\delta x_0^I &= i\bar{\epsilon}\Gamma^I\psi_0 \\
\delta x_{-1}^I &= i\bar{\epsilon}\Gamma^I\psi_{-1} \\
\\
\delta\psi &= -(b_\mu x_0^I + [a_\mu, x^I])\Gamma^\mu\Gamma_I\epsilon - \frac{1}{2}x_0^I[x^J, x^K]\Gamma_{IJK}\epsilon \\
\delta\psi_0 &= 0 \\
\delta\psi_{-1} &= -\text{Tr}(b_\mu x^I)\Gamma^\mu\Gamma_I\epsilon - \frac{1}{6}\text{Tr}([x^I, x^J]x^K)\Gamma_{IJK}\epsilon \\
\\
\delta a_\mu &= i\bar{\epsilon}\Gamma_\mu\Gamma_I(x_0^I\psi - \psi_0x^I) \\
\delta b_\mu &= i\bar{\epsilon}\Gamma_\mu\Gamma_I[x^I, \psi] \\
\delta A_{\mu-1i} &= i\bar{\epsilon}\Gamma_\mu\Gamma_I\frac{1}{2}(x_{-1}^I\psi_i - \psi_{-1}x_i^I) \\
\delta A_{\mu-10} &= i\bar{\epsilon}\Gamma_\mu\Gamma_I\frac{1}{2}(x_{-1}^I\psi_0 - \psi_{-1}x_0^I), \tag{2.13}
\end{aligned}$$

and thus a commutator of dynamical supersymmetry transformations and kinematical ones acts as

$$\begin{aligned}
(\tilde{\delta}_2\delta_1 - \delta_1\tilde{\delta}_2)x^I &= i\bar{\epsilon}_1\Gamma^I\tilde{\epsilon}_2 \equiv \eta^I \\
(\tilde{\delta}_2\delta_1 - \delta_1\tilde{\delta}_2)a^\mu &= i\bar{\epsilon}_1\Gamma^\mu\Gamma_Ix_0^I\tilde{\epsilon}_2 \equiv \eta^\mu \\
(\tilde{\delta}_2\delta_1 - \delta_1\tilde{\delta}_2)A_{-1i}^\mu T^i &= \frac{1}{2}i\bar{\epsilon}_1\Gamma^\mu\Gamma_Ix_{-1}^I\tilde{\epsilon}_2, \tag{2.14}
\end{aligned}$$

where the commutator that acts on the other fields vanishes. Thus, the commutation relation for physical modes is given by

$$\tilde{\delta}_2\delta_1 - \delta_1\tilde{\delta}_2 = \delta_\eta, \tag{2.15}$$

where δ_η is a translation.

If we change a basis of the supersymmetry transformations as

$$\begin{aligned}
\delta' &= \delta + \tilde{\delta} \\
\tilde{\delta}' &= i(\delta - \tilde{\delta}), \tag{2.16}
\end{aligned}$$

we obtain

$$\begin{aligned}
\delta'_2 \delta'_1 - \delta'_1 \delta'_2 &= \delta_\eta \\
\tilde{\delta}'_2 \tilde{\delta}'_1 - \tilde{\delta}'_1 \tilde{\delta}'_2 &= \delta_\eta \\
\tilde{\delta}'_2 \delta'_1 - \delta'_1 \tilde{\delta}'_2 &= 0.
\end{aligned} \tag{2.17}$$

These 32 supersymmetry transformations are summarised as $\Delta = (\delta', \tilde{\delta}')$ and (2.17) implies the $\mathcal{N} = 1$ supersymmetry algebra in eleven dimensions,

$$\Delta_2 \Delta_1 - \Delta_1 \Delta_2 = \delta_\eta. \tag{2.18}$$

3 DLCQ limit

In this section, we will take a DLCQ limit of our model and obtain BFSS matrix theory with finite size matrices as desired.

First, we separate the auxiliary fields b^μ from A^μ and define X^μ by

$$A^\mu = X^\mu + b^\mu. \tag{3.1}$$

We identify space-time coordinate matrices by redefining matrices as follows. By rescaling the eight matrices as

$$\begin{aligned}
X^I &= \frac{1}{T} X'^I \\
X^\mu &= X'^\mu,
\end{aligned} \tag{3.2}$$

we adjust the scale of X^I to that of X^μ . T is a real parameter. Next, we redefine fields so as to keep the scale of nine matrices:

$$\begin{aligned}
X'^p &= X''^p \\
X'^i &= X''^i \\
X'^0 &= \frac{1}{T} X''^0 \\
X'^{10} &= \frac{1}{T} X''^{10}
\end{aligned} \tag{3.3}$$

where $p = 1, 2$ and $i = 3, \dots, 9$. We also redefine the auxiliary fields as

$$b^\mu = \frac{1}{T^2} b'''^\mu. \tag{3.4}$$

DLCQ limit of M-theory consists of a light-cone compactification, $x^- \approx x^- + 2\pi R$, where $x^\pm = \frac{1}{\sqrt{2}}(x^{10} \pm x^0)$, and Lorentz boost in x^{10} direction with an infinite momentum. We define light-cone coordinates as

$$\begin{aligned} X'^0 &= \frac{1}{\sqrt{2}}(X^+ - X^-) \\ X'^{10} &= \frac{1}{\sqrt{2}}(X^+ + X^-) \end{aligned} \quad (3.5)$$

We treat b'''^μ as scalars. A matrix compactification [33] on a circle with a radius R imposes following conditions on X^- and the other matrices Y , which represent X^+ , X''^p , X''^i , b'''^μ , and Ψ :

$$\begin{aligned} X^- - (2\pi R)\mathbf{1} &= U^\dagger X^- U \\ Y &= U^\dagger Y U, \end{aligned} \quad (3.6)$$

where U is a unitary matrix. After the compactification, we cannot redefine fields freely. A solution to (3.6) is given by $X^- = \bar{X}^- + \tilde{X}^-$, $Y = \tilde{Y}$ and

$$U = \tilde{U} \otimes \mathbf{1}_{\text{Lorentzian}}, \quad (3.7)$$

where $U(N)$ part is given by,

$$\tilde{U} = \begin{pmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & & 1 \\ 0 & & & 0 \end{pmatrix} \otimes \mathbf{1}_{n \times n}. \quad (3.8)$$

A background \bar{X}^- is

$$\bar{X}^- = -T^3 \bar{x}_0^- T^0 - (2\pi R) \text{diag}(\cdots, s-1, s, s+1, \cdots) \otimes \mathbf{1}_{n \times n}, \quad (3.9)$$

and a fluctuation \tilde{x} that represents $u(N)$ parts of \tilde{X}^- and \tilde{Y} is

$$\begin{pmatrix} \tilde{x}(0) & \tilde{x}(1) & \cdots & \\ \tilde{x}(-1) & \ddots & \ddots & \\ \vdots & \ddots & & \tilde{x}(1) \\ & & \tilde{x}(-1) & \tilde{x}(0) \end{pmatrix}. \quad (3.10)$$

Each $\tilde{x}(s)$ is a $n \times n$ matrix, where s is an integer. That is, the (s, t) -th block is given by $\tilde{x}_{s,t} = \tilde{x}(s-t)$.

We make a Fourier transformation,

$$\tilde{x}(s) = \frac{1}{2\pi\tilde{R}} \int_0^{2\pi\tilde{R}} d\tau x(\tau) e^{is\frac{\tau}{\tilde{R}}}, \quad (3.11)$$

where $x(\tau)$ is a $n \times n$ matrix in one-dimension and $R\tilde{R} = 2\pi$. From (3.9), (3.10) and (3.11), the following identities hold:

$$\begin{aligned} \sum_t \tilde{x}_{s,t} \tilde{x}'_{t,u} &= \frac{1}{2\pi\tilde{R}} \int_0^{2\pi\tilde{R}} d\tau x(\tau) x'(\tau) e^{i(s-u)\frac{\tau}{\tilde{R}}} \\ \text{tr}(\sum_{s,t} \tilde{x}_{s,t} \tilde{x}'_{t,s}) &= V \frac{1}{2\pi\tilde{R}} \int_0^{2\pi\tilde{R}} d\tau \text{tr}(x(\tau) x'(\tau)) \\ [\bar{x}^-, \tilde{x}]_{s,t} &= \frac{1}{2\pi\tilde{R}} \int_0^{2\pi\tilde{R}} d\tau \partial_\tau x(\tau) e^{i(s-t)\frac{\tau}{\tilde{R}}}, \end{aligned} \quad (3.12)$$

where tr is a trace over $n \times n$ matrices and $V = \sum_s 1$. We will use these identities later.

Next, let us boost the system in x^{10} direction:

$$\begin{aligned} \tilde{X}^+ &= \frac{1}{T} \tilde{X}'''^+ \\ \tilde{X}^- &= T \tilde{X}'''^- \\ \tilde{X}''^p &= \tilde{X}'''^p \\ \tilde{X}'''^i &= \tilde{X}'''^i. \end{aligned} \quad (3.13)$$

IMF limit is achieved when $T \rightarrow \infty$. The second equation implies that $X^- = -T^3 \bar{x}_0^- T^0 + T X'''^-$, where $X'''^- = \bar{X}'''^- + \tilde{X}'''^-$ and $\bar{X}'''^- = -(2\pi R') \text{diag}(\cdots, s-1, s, s+1, \cdots) \otimes \mathbf{1}_{n \times n}$. $R' = \frac{1}{T} R$ goes to zero when $T \rightarrow \infty$. To keep supersymmetry, the fermionic fields need to behave as

$$\Psi = \frac{1}{T} \Psi'''. \quad (3.14)$$

To summarize, relations between the original fields and the fixed fields when $T \rightarrow \infty$ are

$$\begin{aligned}
a^0 &= \frac{1}{\sqrt{2}} \left(\frac{1}{T^2} x'''^+ - x'''^- \right) \\
a^p &= x'''^p \\
x^i &= \frac{1}{T} x'''^i \\
x^{10} &= \frac{1}{\sqrt{2}} \left(\frac{1}{T^3} x'''^+ + \frac{1}{T} x'''^- \right) \\
x_0^i &= \frac{1}{T} x_0'''^i \\
x_0^{10} &= \frac{1}{\sqrt{2}} \left(\frac{1}{T^3} x_0'''^+ + \frac{1}{T} x_0'''^- \right) - \frac{1}{\sqrt{2}} T \bar{x}_0^- \\
b^\mu &= \frac{1}{T^2} b'''^\mu \\
\psi &= \frac{1}{T} \psi''' \\
\psi_0 &= \frac{1}{T} \psi_0''' .
\end{aligned} \tag{3.15}$$

By using these relations, equations of motion of the auxiliary fields b^μ ,

$$b^\mu = \frac{1}{(x_0^I)^2} (-x_0^I [a^\mu, x_I] - \frac{1}{2} E^{\mu\nu\lambda} [a_\nu, a_\lambda] + i \bar{\psi}_0 \Gamma^\mu \psi) \tag{3.16}$$

are rewritten as

$$\begin{aligned}
b'''^0 &= -\frac{2}{(\bar{x}_0^-)^2} [x'''^1, x'''^2] + O\left(\frac{1}{T}\right) \\
b'''^1 &= \left(-\frac{\sqrt{2}}{(\bar{x}_0^-)^2} [x'''^2, x'''^-] + \frac{1}{\bar{x}_0^-} [x'''^1, x'''^-] \right) + O\left(\frac{1}{T}\right) \\
b'''^2 &= \left(\frac{\sqrt{2}}{(\bar{x}_0^-)^2} [x'''^1, x'''^-] + \frac{1}{\bar{x}_0^-} [x'''^2, x'''^-] \right) + O\left(\frac{1}{T}\right).
\end{aligned} \tag{3.17}$$

If we substitute them and (3.15) to the action (2.9), we obtain

$$\begin{aligned}
S &= \frac{1}{T^2} \text{Tr} \left(\frac{1}{2(\bar{x}_0^-)^2} [x'''^-, x'''^p]^2 + \frac{1}{4} [x'''^-, x'''^i]^2 - \frac{1}{2(\bar{x}_0^-)^2} [x'''^p, x'''^q]^2 - \frac{1}{2} [x'''^p, x'''^i]^2 - \frac{(\bar{x}_0^-)^2}{8} [x'''^i, x'''^j]^2 \right. \\
&\quad \left. - \frac{i}{2\sqrt{2}} \bar{\psi}''' \Gamma^0 [x'''^-, \psi'''] - \frac{i}{2} \bar{\psi}''' \Gamma^p [x_p''', \psi'''] - \frac{i}{2\sqrt{2}} \bar{x}_0^- \bar{\psi}''' \Gamma_{10i} [x'''^i, \psi'''] \right) + O\left(\frac{1}{T^3}\right).
\end{aligned} \tag{3.18}$$

Therefore, the action reduces to

$$\begin{aligned}
\hat{S} &= \frac{1}{T^2} \text{Tr} \left(\frac{1}{2(\bar{x}_0^-)^2} [x'''^-, x'''^p]^2 + \frac{1}{4} [x'''^-, x'''^i]^2 - \frac{1}{2(\bar{x}_0^-)^2} [x'''^p, x'''^q]^2 - \frac{1}{2} [x'''^p, x'''^i]^2 - \frac{(\bar{x}_0^-)^2}{8} [x'''^i, x'''^j]^2 \right. \\
&\quad \left. - \frac{i}{2\sqrt{2}} \bar{\psi}''' \Gamma^0 [x'''^-, \psi'''] - \frac{i}{2} \bar{\psi}''' \Gamma^p [x_p''', \psi'''] - \frac{i}{2\sqrt{2}} \bar{x}_0^- \bar{\psi}''' \Gamma_{10i} [x'''^i, \psi'''] \right)
\end{aligned} \tag{3.19}$$

in $T \rightarrow \infty$ limit. By redefining

$$\begin{aligned}
x'''^i &\rightarrow \frac{2^{\frac{1}{4}}\sqrt{T}}{\sqrt{\bar{x}_0}}x'''^i \\
x'''^p &\rightarrow \frac{\sqrt{\bar{x}_0}T}{2^{\frac{1}{4}}}x'''^p \\
x'''^- &\rightarrow 2^{\frac{1}{4}}\sqrt{\bar{x}_0}Tx'''^- \\
\psi''' &\rightarrow \frac{2^{\frac{1}{8}}T^{\frac{3}{4}}}{(\bar{x}_0)^{\frac{1}{4}}}\psi''',
\end{aligned} \tag{3.20}$$

we obtain

$$S = \text{Tr}\left(\frac{1}{2}[x'''^-, x'''^I]^2 - \frac{1}{4}[x'''^I, x'''^J]^2 - \frac{i}{2}\bar{\psi}''' \Gamma^0 [x'''^-, \psi'''] - \frac{i}{2}\bar{\psi}''' \Gamma^p [x_p''', \psi'''] - \frac{i}{2}\bar{\psi}''' \Gamma^{10i} [x_i''', \psi''']\right). \tag{3.21}$$

The background in x'''^- is modified, where $\frac{1}{\sqrt{T}}R' \rightarrow R'$. By using the identities (3.12), we can rewrite (3.21) and obtain the action of BFSS matrix theory with finite n ,

$$S = \int_{-\infty}^{\infty} d\tau \text{tr} \left(\frac{1}{2}(D_0 x^I)^2 - \frac{1}{4}[x^I, x^J]^2 + \frac{1}{2}\bar{\psi} \Gamma^0 D_0 \psi - \frac{i}{2}\bar{\psi} \Gamma^p [x_p, \psi] - \frac{i}{2}\bar{\psi} \Gamma^{10i} [x_i, \psi] \right). \tag{3.22}$$

We have used $\tilde{R}' = \infty$ because $R' \rightarrow 0$ when $T \rightarrow \infty$. In DLCQ limit of our model, we see that X^- disappears and X^+ changes to τ as in the case of the light-cone gauge fixing of the membrane theory.

The way to take DLCQ limit (3.2) - (3.15) is essentially the same as in the case of the Hermitian model [3] because the limit realizes the "novel Higgs mechanism" [34].

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